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Measurement of radial thermal conductivity of a cylinder using a time-varying heat flux method

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ABSTRACT

Despite a variety of techniques available for thermal conductivity measurement, there is a lack of methods to directly measure the radial thermal conductivity of a cylinder. This presents a critical gap in heat transfer metrology, particularly in presence of anisotropic thermal conduction, such as in a Li-ion cell, where radial thermal conductivity plays a key role in determining performance and safety. This paper reports a technique for measurement of radial thermal conductivity of a cylinder by accounting for variable heat flux into the cylinder when heated on the outside. It is shown that heat flux into the cylinder from a thin heater wrapped around its surface varies significantly with time. This variation, which was neglected in past work, is accounted for by developing a variable heat flux model for experimental data analysis. For two different materials, measurements of radial thermal conductivity using this approach are shown to be in close agreement with standard thermal conductivity measurement using the transient plane source method. Radial thermal conductivity of a 26650 Li-ion cell is measured to be 0.39 W/mK. Besides contributing a new approach for thermal metrology in cylindrical systems, this work also improves the understanding of thermal phenomena in Li-ion cells.

1. Introduction

Thermal conductivity and heat capacity are the two key thermo-physical properties that determine the thermal performance of any material, component or system [1,2]. Thermal conductivity of a body is defined based on Fourier law of thermal conduction [2,3]. For most engineering materials, thermal conductivity is isotropic, although in some cases, such as in a Li-ion cell, the value of the thermal conductivity may be strongly direction-dependent [4,5]. A number of experimental methods have been used for measurement of thermal properties of engineering materials, components and systems [4,6–8]. Most of these methods compare experimental measurement of the thermal response to an imposed heat flux with an appropriate analytical model to determine thermal conductivity, and in some cases, heat capacity as well. For example, several methods impose a one-dimensional heat flux through the body of interest and measure temperature difference across the body to determine the thermal conductivity [9]. The effect of thermal contact resistance in such measurements has been accounted for [10]. Laser flash methods impose a heat flux on one face of the test sample and measure the transient thermal response on the other face in order to determine thermal conductivity [6]. Comparison of the short-time thermal response to transient heating of a test sample with an analytical model for thermal conduction in an infinite medium has also been used to measure thermal conductivity [7]. Response to steady-periodic heating has also been used to determine thermal properties of materials [11]. Many of these methods require specific geometries for test samples and are suited for measurement in only specific directions. In particular, several of these methods are inappropriate for measurement of radial thermal conductivity of cylindrical samples. This may not be an important concern for bodies with known isotropy in thermal conductivity, wherein the measurement can be easily carried out in the axial direction instead. However, some cylindrical bodies exhibit anisotropy, making it necessary to directly measure the radial component of thermal conductivity. One-dimensional heat flux methods are not possible to use in such a case, since imposition of a heat flux on the outer radial surface of a cylinder does not produce a steady-state [4], and since measurement of temperature inside a cylinder is often not possible [12–14]. Plane heat source based methods are also not appropriate since a heat source on the outer surface of the cell may result in heat conduction in all three directions, making it impossible to isolate only the radial component. Laser flash based methods are also not appropriate, since these methods require a sample with flat faces for both heat pulse absorption and temperature measurement. Development of methods to measure the radial thermal conductivity is therefore of much interest.

Measurement of radial thermal conductivity is important for

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understanding the thermal safety of cylindrically shaped engineering systems such as a Li-ion cell. Due to heat generation during charging and discharging processes [15], there is significant temperature rise in a Li-ion cell [15–17], which leads to serious safety and performance challenges. Particularly, temperature rise beyond a certain threshold initiates multiple, cascaded exothermic reactions that result in an undesirable thermal runaway scenario involving uncontrolled temperature rise and catastrophic failure [18–20]. Radial thermal conductivity of a cylindrical Li-ion cell is a key thermal property, the value of which is critical for accurate pre-operation design, as well as run-time thermal performance modeling and prediction. Unlike heat capacity, which can be easily measured through calorimetry-based techniques, only a limited amount of literature exists on measurement of this important thermal property [4,5]. Drake et al. [4] have reported radial thermal conductivity measurement by heating the cell on its outer radial surface with a thin heater and measuring the resulting temperature rise as a function of time. Measurements indicated very strong anisotropy, highlighted by a 150X difference in radial and axial thermal conductivities. This measurement was based on an analytical model that assumed constant heat flux into the cell as a function of time. However, it is possible that heat flux may actually vary with time due to heat conduction into the insulation material, and due to heating up of the thin heater material itself. The first effect, while not important for a steady-state measurement, may be significant for a transient measurement such as the one conducted by Drake et al. The second effect can be minimized by reducing the thermal mass of the heater, but cannot be eliminated completely. As a result of these effects, it is necessary to redefine the thermal conductivity measurement methodology by either modifying the analytical model to account for heat losses, or by modifying the experimental conditions in order to minimize or compensate for such heat losses. Such corrections may increase the accuracy of radial thermal conductivity measurements using this method.

This paper presents experimental measurements to demonstrate the presence of significant variation in heat flux as a function of time in experiments for radial thermal conductivity measurements. Further, an approach to correct for this effect is demonstrated through experiments and data analysis. A modified analytical model capable of accounting for time-varying heat flux into the cylinder is presented. Heat flux measurements are combined with optimization of experimental time duration through sensitivity analysis to accurately determine the radial thermal conductivity. It is shown that this approach results in accurate measurement of radial thermal conductivity of two test materials, which are found in both cases to be in good agreement with measurements based on a separate, independent method. This variable heat flux based approach is implemented for measurement of radial thermal conductivity of a 26650 Li-ion cell. Analysis of this method shows significant dependence of radial thermal conductivity on the assumed value of heat capacity.

2. Mathematical model

2.1. Variable heat flux model

Consider a cylindrical body of radius R subjected to a certain time-varying heat flux on its outer surface at $r = R$ due to heat generated in a thin heater wrapped around its outer radial surface. The interest is in predicting temperature rise on the outer surface of the cylinder as a function of time, which may be compared against experimental measurements to determine the thermal properties of the cylinder. Specifically, measurement of the radial thermal conductivity is of interest, since the heat capacity can be easily measured using calorimetry-based techniques. Assuming the radial thermal conductivity, heat capacity and density of the body to be k_r, C_p and ρ respectively, the energy conservation equation governing the temperature rise $\theta(r,t)$ in the cylinder is given by

$$\frac{k_r}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) = \rho C_p\frac{\partial\theta}{\partial t} \tag{1}$$

where r and t are the radial and time coordinates respectively.

Since the cell is being heated on the outer surface, a heat flux boundary condition applies at $r = R$. Drake et al. solved this problem for constant heat flux on the outer surface [4]. However, as shown by experimental data discussed in section 4.1, heat flux into the test samples does not remain constant with time, even if heat is generated at a constant rate in the heater. As a result, it is important to re-derive a solution for equation (1) to account for time-varying heat flux into the cylinder on the outer surface, $Q(t)$, instead of assuming constant heat flux. This complicates the problem somewhat due to a time-varying boundary condition given by

$$\frac{\partial\theta}{\partial r} = \frac{Q(t)}{k_r} \quad \text{at } r = R \tag{2}$$

An additional boundary condition is that

$$\frac{\partial\theta}{\partial r} = 0 \quad \text{at } r = 0 \tag{3}$$

Finally, temperature rise is assumed to be zero at the initial time. Note that in general, the temperature distribution $\theta(r,t)$ may not reach a steady state unless $Q(t)$ becomes zero at large times, which is not the case here, since the heater around the body continues to generate heat. This lack of a steady state is similar to the problem solved by Drake et al. [4], and can be addressed by subtracting from $\theta(r,t)$ the average temperature rise as a function of time, which in this case is given by $\theta_{mean}(t) = \frac{2\cdot\int_0^t Q(\tau)d\tau}{\rho C_p R}$.

Once this transformation is carried out, the remaining problem still involves a time-varying heat flux at the outer surface, but is nevertheless easier to solve as it does have a steady-state. This problem is solved using the method of undetermined parameters by assuming a series solution comprising time-dependent coefficients $c_n(t)$ and eigenfunctions of the corresponding homogeneous problem [21].

$$\theta(r,t) - \theta_{mean}(t) = \sum_{n=1}^{\infty} c_n(t)J_0(\lambda_n r) \tag{4}$$

Where J_0 denotes Bessel function of the first kind of order 0 and the eigenvalues λ_n are obtained from the roots of J_1, the Bessel function of the first kind and of order 1. By inserting the assumed form of the solution, equation (4) into the governing energy equation and using the boundary conditions to simplify, the following ordinary differential equation can be derived for $c_n(t)$

$$c'_n(t) + \alpha_r\lambda_n^2 c_n(t) = \frac{\alpha_r R J_0(\lambda_n R)}{N_n \cdot k_r}Q(t) \tag{5}$$

where N_n are norms of the eigenfunctions [2] and $\alpha_r = k/_{\rho C_p}$ is the radial thermal diffusivity.

Equation (5) can be solved along with the zero initial condition for $c_n(t)$ to result in the following final form of the solution for the temperature rise.

$$\theta(r,t) = \frac{2}{\rho C_p R}\int_0^t Q(\tau)d\tau + \sum_{n=1}^{\infty}\frac{\alpha_r R J_0(\lambda_n R)}{N_n k_r}\int_0^t Q(\tau)\exp(-\alpha_r\lambda_n^2(t-\tau))d\tau \tag{6}$$

Equation (6) represents the analytically derived temperature rise as a function of time that can be compared with experimental data to determine the radial thermal conductivity. This requires measurement of heat flux $Q(t)$ and evaluation of integrals involving $Q(t)$ in equation (6).

For a special case where $Q(t) = Q_0$ is constant, such as the case considered by Drake et al. [4], equation (6) can be simplified to

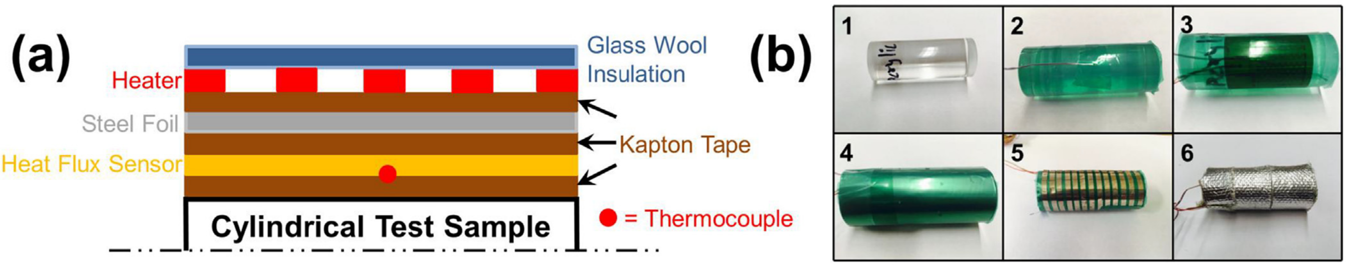

Fig. 1. (a) A schematic of the various thin materials wrapped around the test sample for measurements (not to scale); (b) Various steps in the assembly process for test samples for radial thermal conductivity measurements.

$$\theta(r, t) = \frac{2Q_0 t}{\rho C_p R} + \frac{2Q_0 R}{k_r}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{(\lambda_n R)^2 J_0(\lambda_n R)} - \frac{2Q_0 R}{k_r}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{(\lambda_n R)^2 J_0(\lambda_n R)}\exp(-\alpha_r \lambda_n{}^2 t) \tag{7}$$

The first and third terms in equation (7) match exactly with the result presented by Drake et al. [4] for the special case of constant heat flux. Additionally, it can also be shown through computation that the second term of equation (7) is also the same as the second term of their result. Therefore, the generalized variable heat flux result derived here (equation (6)) correctly reduces to the previous result for the special case of no change in heat flux as a function of time.

When *Q(t)* is constant, as assumed by Drake et al., C_p and k_r appear independently in the first two terms in equation (7) respectively [4]. However, measurements summarized in Section 4.1 show significant time variation in the heat flux entering the cylinder, *Q(t)*, and therefore, equation (6) is a more appropriate analytical solution for the problem. In such a case, *Q(t)* must be directly measured, for example with a heat flux sensor embedded between the thin heater and the cylinder, and used in equation (6). This theoretical model that accounts for time variation in heat flux can be compared with experimental data to determine the values of the thermophysical properties of the body. However, the dependence of the temperature distribution on k_r and C_p is not explicit, as was the case with constant heat flux, and therefore, there is a need to establish a sensitivity analysis to design experiments that accurately determine the thermophysical parameters. Specifically, the interest here is in measuring k_r, since C_p can be easily measured using calorimetry based techniques.

2.2. Sensitivity analysis

Sensitivity analysis of experimental data on temperature and heat flux measurements is carried out. The key parameter of interest to be determined from such an analysis is the optimal time duration of an experiment that results in highest sensitivity of experimental data to the quantity to be determined, in this case, the radial thermal conductivity. The D-Optimality parameter Δ^+, which quantifies such sensitivity is given by Refs. [8,22].

$$\Delta^+ = \frac{1}{\tau}\int_0^{\tau}(X^+)^2(\theta)^{-2}d\tau' \tag{8}$$

Where

$$X^+ = k_r\frac{d\theta}{dk_r} \tag{9}$$

and τ is the experiment duration.

The optimal experimental time duration, τ_{opt} is to be chosen such as to maximize Δ^+ [8,22]. Once the optimal experimental time duration has been determined, the radial thermal conductivity k_r is determined as that value which minimizes the least squares error ε between experimentally measured temperature rise at the outer surface of the cylinder and the theoretical model over the experimental time duration, given by

$$\varepsilon = \sqrt{\frac{1}{\tau_{opt}}\int_0^{\tau_{opt}}(\theta_{\exp} - \theta(R, \tau))^2 d\tau} \tag{10}$$

where $\theta_{\exp}$ is the experimentally measured temperature rise and θ is the temperature rise predicted by the analytical model, equation (6).

It is important to note that evaluating X^+ in equation (9) requires differentiating equation (6), due to which, k_r appears explicitly in the expression for X^+, and therefore Δ^+. Consequently, determining the optimal experimental time duration using D-Optimality theory itself requires a value for the radial thermal conductivity. An iterative approach is utilized in order to resolve this, wherein an initial value of k_r is assumed, based on which the optimal experimental time duration is determined. Experimental data for this time period are then compared against the theoretical model to determine the value of k_r, which is then used to re-evaluate the optimal experimental time period. This iterative process is repeated until there is minimal change in the value of k_r between successive iterations.

3. Experimental methods and materials

Experiments are carried out on cylindrical test samples of well-known plastic materials, including delrin and acrylic as well as on a 26650 Li-ion cell. Each test sample has a height and diameter of 65 mm and 26 mm respectively. Independent measurements of thermal conductivity of the delrin and acrylic samples are carried out using the transient plane source method for validation of the variable heat flux model.

Fig. 1(a) shows a schematic of the various thin materials around the test sample. Fig. 1(b) shows pictures of various steps in the assembly process of test samples. To start with, the sample is wrapped with 65 μm thick electrically insulating Kapton tape (McMaster, Inc.). A T-type thermocouple (Omega Inc.) with 0.25 mm diameter tip and 180 μm thick heat flux sensor (Omega Inc.) are placed next, followed by a 25 μm thick steel foil (McMaster, Inc.) insulated on both sides with Kapton tape. The steel foil facilitates heat spreading and spatially uniform heat flux into the sample. This may be important because the thin film heater comprises serpentine metal heaters, which may cause significant spatial sensitivity of thermocouple measurements unless thermal spreading is ensured. A 125 μm thick thin film heater (Omega Inc.) is then wrapped around the test sample. Finally, the entire assembly is wrapped with 115 μm thick glass wool insulation (McMaster, Inc.). The axial ends are insulated to minimize axial heat conduction. For experiments where the outgoing heat flux is also of interest, an additional heat flux sensor is placed between the thin film heater and glass wool insulation.

The entire test sample is then placed inside a vacuum chamber customized for routing out electrical and thermocouple wires. Pressure inside the vacuum chamber is maintained at −45 kPa (g) or lower in order to minimize convective heat losses. DC power is supplied to the thin film heater by a Keithley 2410 A power supply. A Keithley 2100 A multimeter measures voltage drop across the heater coil. Thermocouple

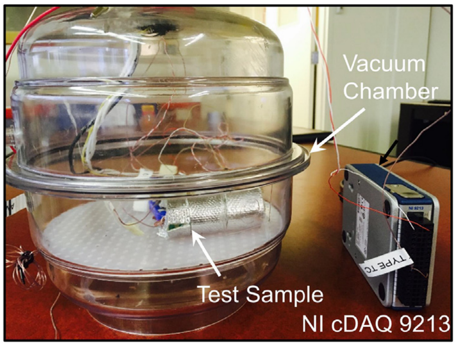

Fig. 2. Picture of the experimental setup.

and heat flux sensor wires are connected to a cDAQ 9213 data acquisition module (National Instruments). Data acquisition is carried out through LabView software operating on a 64-bit personal computer. Heating current sourced from the power supply is also controlled through LabView. Constant heat generation is maintained during experiments. Heat flux into the cell is allowed to vary with time, and is measured by the heat flux sensor. Fig. 2 shows a picture of the experimental setup. The flow of data during the experiment is shown schematically in Fig. 3.

For validating these measurements, a separate, independent measurement of thermal conductivity is also carried out. This independent measurement is based on the transient plane source (TPS) method [7], wherein a thin film heater is sandwiched between two identical samples. A small amount of heat generated in the heater due to Joule heating conducts into the samples. Temperature rise at the interface, measured through resistive thermometry of the thin film heater is compared with a well-established analytical model for thermal conduction in an infinite medium to determine the thermal conductivity and heat capacity of the material. Since the samples are isotropic, thermal conductivity measured in this manner is the same as the radial thermal conductivity of cylindrical samples of the same material. In addition to thermal conductivity, these measurements also help determine the value of heat capacity of the test samples.

4. Results and discussion

4.1. Heat flux measurements

Experimental measurements of radial thermal conductivity by Drake et al. assumed that heat flux into the cylindrical cell during thermal conductivity measurement remains constant with time, based on which, a constant heat flux model was used for data analysis [4].

Experiments are first carried out to test the accuracy of this assumption. A heat flux sensor is placed between the thin film heater and the surface of the cylinder, so that heat flux into the cell is measured directly and compared against the rate of heat generation in the heater. Fig. 4(a) plots the measured heat flux into a 26 mm diameter acrylic cylinder with 0.60 W heating power in the thin film heater. For reference, heat generation rate in the heater is also shown as a broken line. The heat flux is measured directly from the thin film heat flux sensor, whereas the heat generation rate is measured by determining the rate of Joule heating from the measured electric current and potential difference across the cell. Fig. 4(a) clearly shows that the actual heat flux into the sample is lower than the rate of heat generation in the heater. Heat flux into the cell changes significantly as a function of time, rising at first, and then reducing with time. This demonstrates that an assumption of constant heat flux into the cell is not accurate even if a

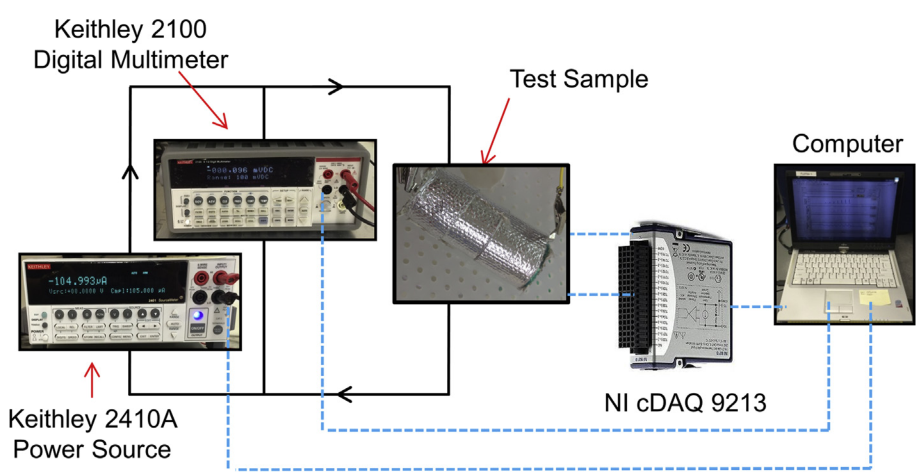

Fig. 3. Schematic of data acquisition and instrument control during experiments.

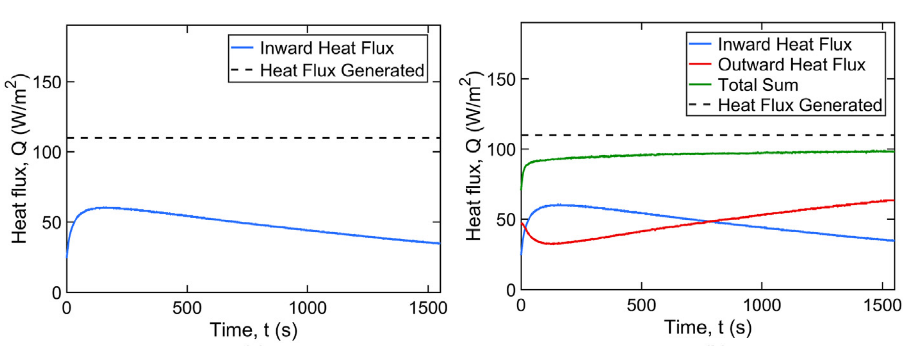

Fig. 4. (a) Inward heat flux as a function of time during an experiment where constant heating occurs in the thin film heater, (b) Inward and outward heat fluxes, as well as their sum as functions of time in this experiment. In both cases, the heat generated in the thin film heater in shown for comparison.

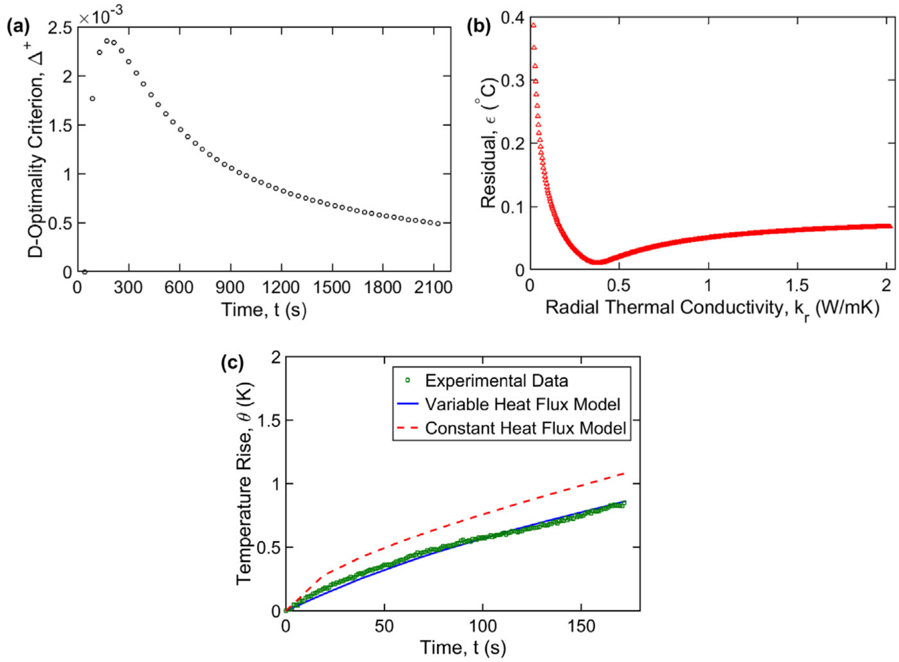

Fig. 5. Results from variable heat flux measurement method for delrin test sample: (a) D-Optimality criterion Δ^+ as a function of experimental time duration, (b) Residual, ε as a function of radial thermal conductivity k_r, and (c) comparison of experimentally measured and theoretically predicted temperature rise as a function of time based on the variable heat flux model. For comparison, theoretical prediction based on the uniform heat flux model is also shown.

constant amount of heat generation occurs in the heater, and that an accurate heat transfer analysis for this measurement must account for this time dependence, such as the analysis presented in section 2.1.

To investigate this further, another experiment is carried out with two heat flux sensors embedded on both sides of the thin film heater. Together, these sensors measure the rate of heat flow from the heater in both directions – radially inwards into the test sample and radially outwards into the outer insulation. Fig. 4(b) plots the measured heat flux by both sensors as functions of time. The total sum, as well as the heat generation rate in the heater are also shown. Fig. 4(b) clearly shows that a non-zero amount of heat flows radially outwards, which varies as a function of time, just like the heat flux into the test sample does. Fig. 4(b) shows that the sum of heat fluxes in both directions is close to the total rate of heat generation in the heater. The small difference between the two diminishes with time, and is likely due to heat absorption by the materials of the heater as well as the two heat flux sensors.

These experiments demonstrate the importance of accounting for time-dependence of heat flux into the cylinder even with constant heat generation in the thin film heater. An analytical model such as one presented in Section 2.1 is more appropriate for data analysis of these experiments compared to constant heat flux models presented in the past. Alternately, heat generation rate in the thin film heater must be dynamically modulated in order to hold the heat flux constant with time, so that the constant heat flux model proposed by Drake et al. remains valid. The latter is a more difficult approach, requiring the use of control algorithms to maintain a constant heat flux. This is particularly difficult for experiments where sensitivity analysis results in short optimal experimental time duration. Therefore, the variable heat flux approach is followed in experiments, described next, for determining radial thermal conductivity of acrylic and delrin cylinders. In each case, sensitivity analysis is carried out to determine the optimal experimental duration in an iterative fashion.

4.2. Sensitivity analysis and k_r measurements on delrin

Results from sensitivity analysis carried out for experiments on a delrin test sample are summarized in Fig. 5. The D-optimality criterion, equation (8) is plotted as a function of time in Fig. 5(a), showing a peak at $\tau_{opt} = 173$ s. The optimal experimental time duration is determined using the iterative method described in section 2.2. 5–6 iterations are found to be sufficient for convergence of k_r for these and subsequent experiments. At the optimal experimental time duration, the residual ε, given by equation (10) is plotted as a function of k_r in Fig. 5(b). This plot shows a minima for ε at a specific value of k_r, at which there is closest agreement between experiments and theoretical model, and which therefore, is the experimentally determined value of the radial thermal conductivity. As summarized in Table 1, the value of $k_r = 0.38$ W/mK measured in this manner is very close to the value of 0.42 W/mK determined through independent measurement based on the transient plane source method.

Fig. 5(c) plots the experimentally measured temperature rise, θ_{exp} and prediction from the theoretical model, θ_{th} as functions of time for the k_r value determined in this manner. There is good agreement between the two, indicating that the experimental measurements and data analysis procedure accurately determine the radial thermal

Table 1
Measured values of radial thermal conductivity in W/mK of delrin, acrylic and 26650 Li-ion cell based on variable heat flux measurement method. For comparison, results from measurements based on the transient plane source method are also listed for delrin and acrylic. Values of heat capacity for these materials, measured separately, are also listed. In the case of Li-ion cell, the measured thermal conductivity refers to the radial component.

	Thermal Conductivity: Variable Heat Flux Method (W/mK)	Thermal Conductivity: Transient Plane Source Method (W/mK)	Heat Capacity (determined separately) J/kgK
Delrin	0.38	0.42	1496
Acrylic	0.22	0.26	1367
Li-ion cell	0.39	–	800

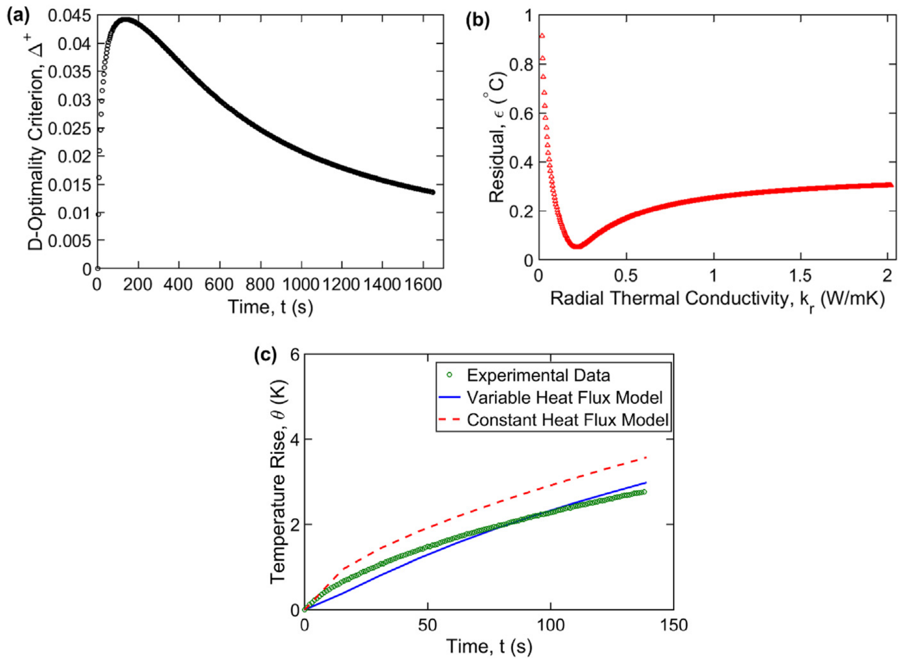

Fig. 6. Results from variable heat flux measurement method for acrylic test sample: (a) D-Optimality criterion Δ^+ as a function of experimental time duration, (b) Residual, ε as a function of radial thermal conductivity k_r, and (c) comparison of experimentally measured and theoretically predicted temperature rise as a function of time based on the variable heat flux model. For comparison, theoretical prediction based on the uniform heat flux model is also shown.

conductivity by accounting for the time variation of heat flux. For comparison, Fig. 5(c) also plots the expected temperature curve if the time-dependence of heat flux is not accounted for. This curve deviates significantly from the experimentally measured temperature curve, which indicates the importance of appropriately accounting for time variation in heat flux through the analytical model presented in section 2.

4.3. Sensitivity analysis and k_r measurements on acrylic

Similar measurements and analysis are carried out on a test sample made of acrylic. Fig. 6(a) and (b) plot the D-optimality parameter Δ^+ as a function of experimental time duration, and the residual ε as a function of k_r for the optimal experimental time duration respectively. Similar to the delrin case, a peak in D-optimality parameter and a minima in the residual are observed. The value of the optimal experimental time duration is found to be 139 s. As summarized in Table 1, the value of thermal conductivity determined in this fashion, 0.22 W/mK, is close to the transient plane source method based measurement of 0.26 W/mK. Fig. 6(c) compares experimentally measured and theoretically determined variation of temperature rise as a function of time for acrylic, showing good agreement over the entire experimental duration. Similar to the previous case, Fig. 6(c) shows significant deviation between experimental measurement and theoretical model that does not account for time variation in heat flux.

These measurements demonstrate that the analytical model developed in Section 2 correctly accounts for time-varying heat flux in these experiments. Good agreement of measured radial thermal conductivity with an independent measurement thermal conductivity for two different materials establishes the accuracy of the measurement method.

4.4. Measurements on 26650 Li-ion cell

Radial thermal conductivity is a critical thermophysical property of a Li-ion cell that affects its thermal performance in both nominal and thermal runaway conditions [16–18]. Radial thermal conductivity measurements are carried out on a 26650 Li ion cell using the time-varying heat flux method. Experimental results are summarized in Fig. 7. Similar to previously discussed data on acrylic and delrin samples, a peak in Δ^+ is observed at $\tau_{opt} = 89$ s when plotted as a function of time, and a minima in the residual ε is observed, which determines the value of the radial thermal conductivity. The radial thermal conductivity of the cell is found to be 0.39 W/mK. Past measurements that assumed constant heat flux in similar experiments reported somewhat lower value, likely because of not having accounted for variable heat flux that occurs in these experiments.

Direct measurement of radial thermal conductivity of a Li-ion cell is critical because of well-known anisotropy in thermal conductivity of a Li-ion cell [4,5]. While in other isotropic materials, the radial thermal conductivity can be determined indirectly by measuring the axial thermal conductivity instead, in the case of an anisotropic material such as a Li-ion cell, a direct and accurate measurement of the radial thermal conductivity is not straightforward and can not be measured using standard thermal conductivity measurement techniques. The measurement methods described here address a critical lack of measurement methods for radial thermal conductivity.

4.5. Importance of heat capacity and uncertainty analysis

In all experiments described here, the heat capacity of the sample is assumed to be known, and is determined through separate, independent measurements. Since the thermal response of test samples is in general a function of both thermal conductivity and heat capacity, data analysis is carried out to determine the importance of accurate information about the heat capacity. A number of different values are assumed for the heat capacity of delrin, and data from variable heat flux experiments are re-analyzed in order to determine the radial thermal conductivity for each assumed value of heat capacity. Results summarized in Fig. 8 indicate strong dependence of the determined value of the radial thermal conductivity on the assumed value of heat capacity. In general, the value of k_r reduces as the assumed value of C_p increases. This demonstrates the importance of accuracy in the assumed value of heat capacity. In this work, the heat capacity of delrin and acrylic is measured independently using the transient plane source method. This method, described previously in section 3, is based on measurement of temperature rise as a function of time in a thermally infinite sample being heated by a plane heat source and comparison of experimental data with an analytical model for thermal conduction in an infinite medium to determine the thermal conductivity and heat capacity of the sample [7]. This method measures heat capacity with an accuracy of

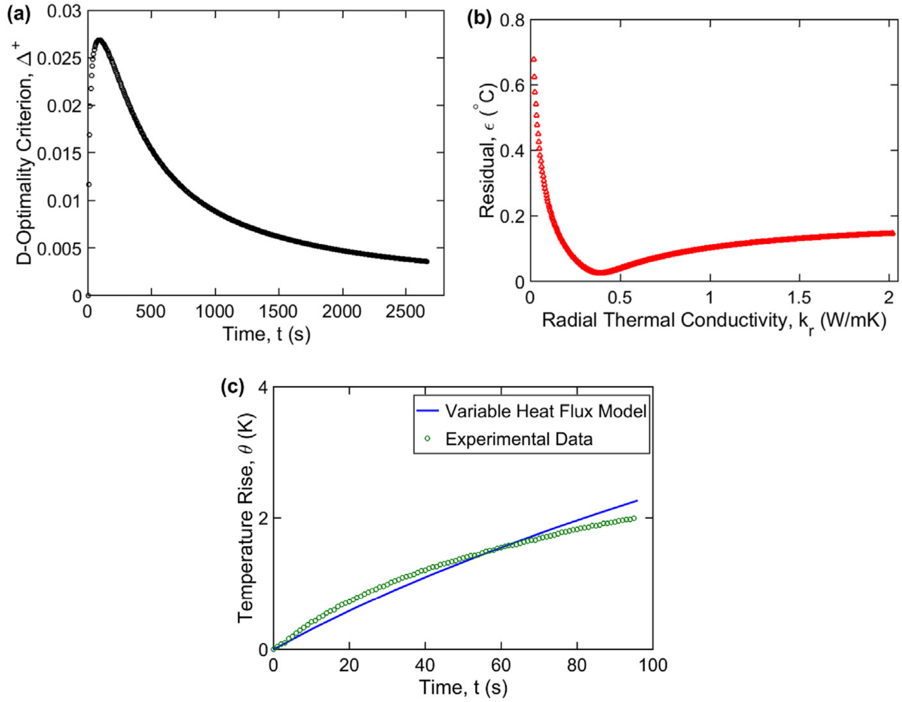

Fig. 7. Results from variable heat flux measurement method for a 26650 Li-ion cell: (a) D-Optimality criterion Δ^+ as a function of experimental time duration, (b) Residual, ε as a function of radial thermal conductivity k_r, and (c) comparison of experimentally measured and theoretically predicted temperature rise as a function of time based on the variable heat flux model.

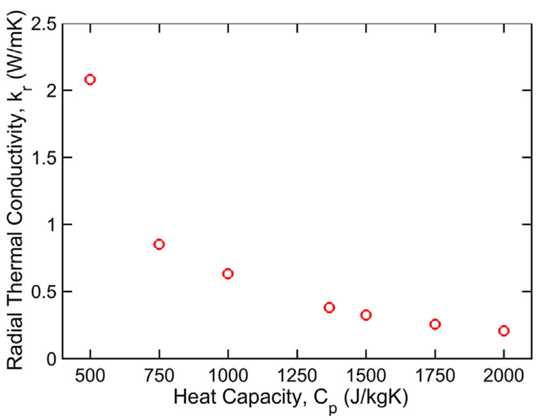

Fig. 8. Measured radial thermal conductivity as a function of heat capacity for variable heat flux measurements on delrin test sample.

around ± 5%, and therefore, is not expected to introduce large uncertainty in k_r measurement. In the case of a Li-ion cell, the transient plane source method is not possible to be used due to thermal conduction anisotropy in the cell, and therefore, the heat capacity is measured through a calorimetry approach, wherein the cell is heated up and placed in a bath of cold water in an insulated vessel. Temperatures of water and cell are measured as functions of time. The temperature change at steady state is used in an energy balance to determine the unknown heat capacity of the cell. This method has been used recently for heat capacity measurement of an 18650 Li ion cell [20].

Among the various sources of uncertainty in measurement of radial thermal conductivity, uncertainty in heat capacity, measured through other methods, is the largest. Based on Fig. 8, in the heat capacity range of interest, a 5% uncertainty in heat capacity measurement results in around 7% uncertainty in thermal conductivity measurement. Other sources of experimental uncertainty are relatively minor, and include uncertainty in various electrical measurements such as voltages induced in heat flux sensor and thermocouples, uncertainty in the heating current, and change in electrical resistivity of the heater with temperature. Voltage measurement instruments used in these experiments offer very low uncertainty. Further, temperature rise in the experiments is kept low enough to not significantly change the heater resistivity. On the overall, therefore, a 7% uncertainty in measurement of radial thermal conductivity is estimated.

5. Conclusions

While a number of experimental methods are available for thermal conductivity measurements in general, there is a lack of methods for direct measurement of the radial thermal conductivity of a cylinder. This can be a particularly important concern in the case of materials with anisotropic thermal conduction where the radial thermal conductivity is not equal to the axial thermal conductivity that is easier to measure. While past work has presented a method for radial thermal conductivity measurement through heat flux imposed on the cylinder's outer surface, this paper shows the importance of accounting for variation in the heat flux as a function of time. A new theoretical model is developed for data analysis that accounts for such variation. Experiments carried out in this work result in good agreement of the measured radial thermal conductivity with independent measurements for two different materials. The theoretical model and experimental data presented here improve our understanding of thermal metrology, particularly for measuring the radial thermal conductivity of cylindrical bodies, which is very important for several engineering applications, such as Li-ion cells.

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